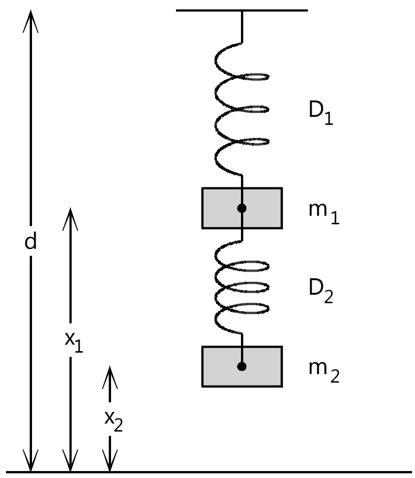
Exercise Sheet 2

Mass-Spring-Double-Pendulum



This sheet is based on exercise sheet 1.

In this task, a double pendulum is to be simulated. The pendulum consists of two masses (m_1 and m_2) and two springs (D_1 and D_2). Springs and masses are interconnected according to the figure above. At its upper end D_1 is connected wit a fixed suspension point. The suspension point is located at a height of $d = 2\,m$ above the ground. L $m_1 = 0{,}015 kg$ et $m_2 = 0{,}01\,kg$ and . Both springs are linear springs with $D_1 = 0{,}4\frac{N}{m}$ and $D_2 = 0{,}3\frac{N}{m}$. The rest length of the upper spring is , t $l_{0,1} = 0{,}1\,m$ he rest length of the lower spring is $l_{0,2} = 0{,}08\,m$. For sake of simplicity, it is assumed that the masses are point-shaped. Let us assume that the earths gravitational acceleration be $g = 9{,}81\frac{m}{s^2}$.

As in exercise sheet 1, the positions of the two masses x_1 and x_2 are measured from the ground. At the beginning of the simulation, the mass m_1 is at a height of $1{,}4m$ above the ground and the mass m_2 is at a height of $0{,}8m$ above the ground. The speeds of the two masses are v_1 and v_2 . An upward speed shall have a positive sign, a downward speed a negative sign. At the beginning, the speed of both masses is $0\frac{m}{s}$.

There are spring forces and gravitational forces applying to the masses. Let F_1 and F_2 be the total forces applying to m_1 and m_2 , respectively. Let a_1 and a_2 be the accelerations due to these forces. For the variables x_1 , x_2 , v_1 , v_2 , F_1 , F_2 , a_1 and a_2 upward directed values shall be represented by positive numbers and downward directed values by negative numbers.

As in exercise sheet 1, exercise 6, the flow resistance F_L shall also be taken into account.

$$F_L = \frac{1}{2} A c_w \rho v^2$$

Both masses shall be balls wit are spherical surface. Let r_1 and r_2 be the radii of the two balls with $r_1 = 0{,}1\,m$ and $r_2 = 0{,}1\,m$. The flow resistance coefficient is $c_w = 0{,}15$ for both balls. The mass density of air is $\rho = 1{,}2041 \frac{kg}{m^3}$.

Exercise 1

Provide equations for determining F_1 , F_2 , a_1 and a_2 !

Exercise 2

As in exercise 1, enter the physical variables and formulas to the following table.

Constants

name	*unit*	*value*

State Variables

name	*unit*	*initial value*	*derivation*

Dependent Variables

name	*unit*	*formula*

Exercise 3

The physical system is now to be programmed. The Java class shall be named *MassSpringDoublePendulum.* Apply the concept from exercise 1 to implement this class using the information from the table above.

The plotter shall draw the course of the variables x_1, x_2, v_1 and v_2. Find appropriate ranges for the plotter's x- and y-axis.

Exercise 4

Load the physical system to the Physolator and start the simulation! Execute several simulations and vary the spring constants, the masses of the bodies and the initial positions and velocities of the bodies!

Exercise 5

Double pendulums have a “chaotic” behavior. Other than the plots from the ordinary pendulum in exercise sheet 1, the courses of the variables from doubles pendulums show an irregular behavior. It is hard to describe such a behavior using mathematical formulas.

What happens, if the mass of m_1 is very small? Give it a try and run a simulation with a very small value for m_1 ! Explain your observations!